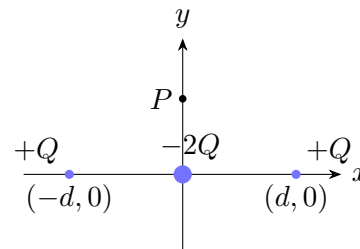


1. Three charges that are fixed and arranged as shown in the figure is one type of electric quadrupole.



- [8 Pts.] (a) Find the expression for the potential at point $P = (0, y)$ as a function of y in terms of the given parameters.

$$V(0, y) = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{d^2 + y^2}} + \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{d^2 + y^2}} - \frac{1}{4\pi\epsilon_0} \frac{2Q}{y}$$

$$V(0, y) = \frac{Q}{2\pi\epsilon_0} \left(\frac{1}{\sqrt{d^2 + y^2}} - \frac{1}{y} \right)$$

- [7 Pts.] (b) What is the total potential energy of the system of these three charges?

$$U = \frac{1}{4\pi\epsilon_0} \left(\frac{Q^2}{2d} - 2 \frac{2Q^2}{d} \right) = -\frac{7Q^2}{8\pi\epsilon_0 d}$$

- [10 Pts.] (c) If the $-2Q$ charge is given an initial velocity v_0 in the positive y direction, what is the minimum value of v_0 such that the charge escapes to a point infinitely far away from the two positive charges?

Energy is conserved $E_i = E_f$

$$U_i = -2QV_i(0, 0) = -2Q \frac{1}{4\pi\epsilon_0} \frac{2Q}{d}$$

and $U_f = 0$. Then, $K_i + U_i = K_f + U_f$

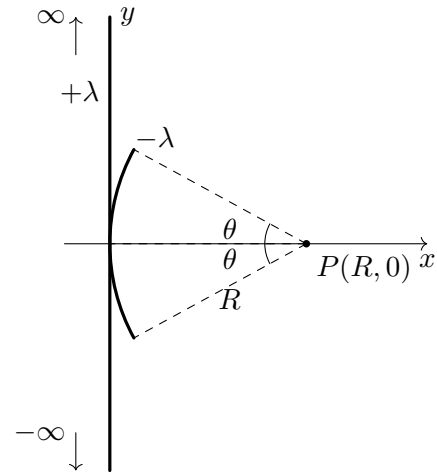
$$\frac{1}{2}mv_0^2 - \frac{Q^2}{\pi\epsilon_0 d} = 0 \quad \Rightarrow \quad v_0 = \sqrt{\frac{2Q^2}{\pi\epsilon_0 dm}}$$

Two thin insulating rods carry uniform line charge densities of equal magnitude λ .

- A straight rod with line charge density $+\lambda$ lies along the y -axis, extending from $y = -\infty$ to $y = +\infty$.
- A second rod with line charge density $-\lambda$ is bent into a circular arc of radius R . The arc is centered at the point $P = (R, 0)$, is symmetric with respect to the x -axis, and passes through the origin. The total arc length is L (so it subtends a total angle L/R).

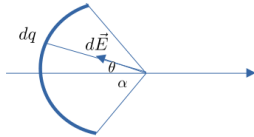
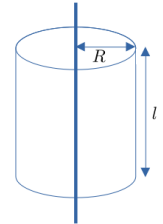
[25 Pts.] 2.

Find the total electric field vector $\vec{E}$ at the point $P = (R, 0)$ (the center of curvature of the arc). Express your answer in terms of λ , L , R , and ϵ_0 . You may assume $L < \pi R$.



Gauss's Law for an infinite rod, $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$

$$|\vec{E}|(2\pi Rl) = \frac{\lambda l}{\epsilon_0} \Rightarrow |\vec{E}|_{rod} = \frac{\lambda}{2\pi\epsilon_0 R}, \quad \hat{i} \text{ direction}$$



For the arc, no y-component due to symmetry

$$dq = -\lambda R d\theta \Rightarrow |d\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{\lambda R d\theta}{R^2}, \text{ and } dE_x = -\frac{1}{4\pi\epsilon_0} \frac{\lambda d\theta}{R} \cos \theta$$

$$E_{x,arc} = -\int_{-\alpha}^{\alpha} \frac{1}{4\pi\epsilon_0} \frac{\lambda d\theta}{R} \cos \theta = -\frac{\lambda}{4\pi\epsilon_0 R} 2 \sin \alpha$$

Since $L = 2R\alpha$

$$E_{x,arc} = -\frac{\lambda}{2\pi\epsilon_0 R} \sin \left(\frac{L}{2R} \right)$$

Then, the total electric field at the point P is

$$\vec{E}(P) = \frac{\lambda}{2\pi\epsilon_0 R} \left(1 - \sin \left(\frac{L}{2R} \right) \right) \hat{i}$$

3. A nonconducting sphere of radius r_0 is uniformly charged with volume charge density ρ_E . It is surrounded by a concentric metal (conducting) spherical shell of inner radius r_1 and outer radius r_2 , which carries a net charge $+Q$. Determine the resulting electric field magnitude in the regions

[6 Pts.] (a) $0 < r < r_0$,

Applying Gauss Law with a spherical Gaussian surface with radius r , $Q_{in} = \rho_E \frac{4}{3}\pi r^3$,

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}, \quad \vec{E} \parallel d\vec{A}$$

$$|\vec{E}|4\pi r^2 = \rho_E \frac{4}{3}\pi r^3 \Rightarrow |\vec{E}| = \frac{\rho_E}{3\epsilon_0}r$$

[6 Pts.] (b) $r_0 < r < r_1$

Now, $Q_{in} = \rho_E \frac{4}{3}\pi r_0^3$,

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}, \quad \vec{E} \parallel d\vec{A}$$

$$|\vec{E}|4\pi r^2 = \rho_E \frac{4}{3}\pi r_0^3 \Rightarrow |\vec{E}| = \frac{\rho_E}{3\epsilon_0} \frac{r_0^3}{r^2}$$

[6 Pts.] (c) $r_1 < r < r_2$, and

Inside a conductor, $|\vec{E}| = 0$,

$Q_{in} = 0$ on the interior surface, induced charge is equal to $-\rho_E \frac{4}{3}\pi r_0^3$

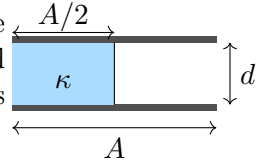
[7 Pts.] (d) $r > r_2$ where the radial distance r is measured from the center of the nonconducting sphere.

Total charge is equal to the sum of the charge of the sphere and charge of the shell,
 $Q_{in} = \rho_E \frac{4}{3}\pi r_0^3 + Q$,

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}, \quad \vec{E} \parallel d\vec{A}$$

$$|\vec{E}|4\pi r^2 = \rho_E \frac{4}{3}\pi r_0^3 + Q \Rightarrow |\vec{E}| = \frac{1}{4\pi\epsilon_0 r^2} \left(Q + \frac{4\pi\rho_E r_0^3}{3} \right)$$

4. A parallel-plate capacitor has a plate separation d and a total plate area A . A linear dielectric slab of relative permittivity κ is inserted so that it completely spans the gap between the plates but covers exactly half the plate area (as in the figure). Fringing is negligible.



- [10 Pts.] (a) Determine the equivalent capacitance C .

Two capacitors in parallel, $C_1 = \kappa\epsilon_0 \frac{A/2}{d}$ and $C_2 = \epsilon_0 \frac{A/2}{d}$,

$$C_{eq} = (1 + \kappa)\epsilon_0 \frac{A}{2d}$$

- (b) Suppose the dielectric is slowly pulled completely out of the capacitor by an external force while the capacitor is connected to a battery that maintains a constant potential difference V_0 .

- [5 Pts.] i. Find the change in the stored energy in the capacitor.

The capacitance after removal, $C_f = \epsilon_0 \frac{A}{d}$,

$$U_i = \frac{1}{2}C_{eq}V_0^2 = \frac{1}{2}(1 + \kappa)\epsilon_0 \frac{A}{2d}V_0^2$$

$$U_f = \frac{1}{2}C_fV_0^2 = \frac{1}{2}\epsilon_0 \frac{A}{d}V_0^2$$

Then,

$$U_f - U_i = \frac{1}{2}\epsilon_0 \frac{A}{d}V_0^2 \left(1 - \frac{1 + \kappa}{2}\right) = \frac{1}{2} \frac{(1 - \kappa)}{2} \epsilon_0 \frac{A}{d}V_0^2$$

- [5 Pts.] ii. Find the work done by the battery.

The charge after removal, $Q_f = CV_0 = \epsilon_0 \frac{A}{d}V_0$,

$$Q_i = C_{eq}V_0 = \frac{(1 + \kappa)}{2}\epsilon_0 \frac{A}{d}V_0 \Rightarrow \Delta Q = \epsilon_0 \frac{A}{d}V_0 \left(1 - \frac{1 + \kappa}{2}\right) = \epsilon_0 \frac{A}{d}V_0 \frac{(1 - \kappa)}{2}$$

Then,

$$W_b = \Delta QV_0 = \epsilon_0 \frac{(1 - \kappa)}{2} \frac{A}{d}V_0^2$$

- [5 Pts.] iii. Find the work done by the external force.

$$U_f = U_i + W_b + W_{ext} \Rightarrow W_{ext} = (U_f - U_i) - W_b,$$

$$W_{ext} = \frac{1}{2}\epsilon_0 \frac{(\kappa - 1)}{2} \frac{A}{d}V_0^2$$